BELAVKIN–KOLOKOLTSOV WATCH–DOG EFFECTS IN INTERACTIVELY CONTROLLED STOCHASTIC COMPUTER-GRAPHIC DYNAMICAL SYSTEMS. A SUMMARY OF MATHEMATICAL RESEARCHES.

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Abstract. This paper contains a summary of mathematical researches of stochastic properties of the long time behavior of a continuously observed (and interactively controlled) quantum–field top. Applications to interactively controlled stochastic computer-graphic dynamical systems are also discussed.

I. INTRODUCTION

The main difficulty to account the high–frequency eye tremor in *mobilevision* (*MV*) [Ju2, Ju3, Ju4, Ju5] is that in this case a solution of the complete MV evolution equations in real time requests about 10^8–10^9 arithmetical operations per second (moreover, it needs special displays of a high refreshing rate ($\sim$ 300–500 frames per second) and a small image inertia). Such account may be performed only on a narrow class of computers for the purposes of scientific experiments on peculiarities of human vision in interactive computer-graphic systems [Ju2, Ju3], but it is very inconvenient for an assimilation of MV as a computer-graphic tool f.e. for an interactive visualization of 2D quantum field theory [Ju5]. So one should to use some stochastic simulation of the interactive processes, i.e. to consider an imitated stochastic process instead of the tremor. Such approach leads to *stochastic mobilevision* (*SMV*) [Ju2], which evolution equations have a stochastic Belavkin–type form [Be, BK, Ko1]. It seems that the interactive effects for ordinary MV and SMV are similar in general, because the interactive processes accounting saccads are not stochastized; though it is not an undisputable fact that they are always identical (f.e. in a situation of the so–called "lateral vision"). The combination of MV with cluster and spline techniques allows to work on computers with 10^5–10^6 arithmetical operations per second (as well as to use simpler devices for eye motion detection and a wider class of displays), whereas all enumerated above circumstances make the tremor accounting in terms of ordinary MV almost unreasonable nowadays.

Nevertheless, all these advantages of SMV are not crucial in view of the permanent progress in the computer hightech (for example, the using of a distributed parallel processing allows to diminish the request for the tremor accounting ordinary MV to $\sim 10^6$ arithmetical operations per second, etc.). A deeper advantage of SMV is more fundamental — it is a presence of the Belavkin–Kolokoltsov watch–dog effects ([Ko2], see also the original papers [CSM, MS] where "watch–dog effects" or a "quantum Zeno paradox" were put under a consideration) in SMV in certain

rather natural and general cases (i.e. for certain values of internal parameters measuring the degree of localization of interaction) that means an *a priori* finiteness of sizes of stochastic "cores" of an image during observation, moreover they may be diminished to several pixels by a suitable choice of a *free* controlling parameter (the so–called *"accuracy of measurement"* [Be, BK, Ko1, Ko2]). The watch–dog effect may be considered as a weaker but also tamer form of nondemolition than the quasistationarity [Ju2, Ju3]: there exists a wide class of models, in which the first is observed whereas the least is broken, one may consider canonical projective G–hypermultiplets [Ju3] (see also [Ju4]) as a simple example.

Thus, a transition from MV to SMV partially solves *a priori* the main problem of dynamics in interactive psychoinformation computer-graphic systems [Ju2, Ju3] — a problem of the nondemolition of images by the interactive processes (i.e. their stability under observation). Certainly, SMV does not solve the nondemolition problem completely *a priori*. It only garantees that the stochastic cores of image have finite sizes during observation, it means that details of image do not diffuse. Nevertheless, they may move, being ruled by the slow eye movements. So though details of image are perserved, the image may be destructed as a whole. It seems that the quasistationarity conditions [Ju2, Ju3] are realistic complements to watch–dog effects and together they provide a complete long–time nondemolition of images.

Also it should be marked that such *a priori* nondemolition in SMV confirms a presence of *a posteriori* one in the tremor accounting ordinary MV.

The purpose of this note is to investigate the Belavkin–Kolokoltsov watch–dog effects in SMV mathematically.

Summarizing arguments above one may conclude that such investigations are motivated by the overlapping of two problems:

1) the difficulty to account the high–frequency eye tremor in ordinary mobilevision, which leads to the necessity to consider tremor's stochastic simulations;
2) the main problem of dynamics in interactive psychoinformation computer-graphic systems, i.e. a problem of the nondemolition of images by the interactive processes; it motivates investigations of long–time properties of (nonlinear) stochastic dynamics in SMV.

So the first problem explains, why stochastic mobilevision is put under a consideration, the second one explains a choice of questions, which are tried to be solved in the paper.

II. MATHEMATICAL ASPECTS OF MOBILEVISION [Ju4, Ju5]

This paragraph is devoted to a brief exposition of mathematical (geometric) aspects of ordinary mobilevision. It may be omitted by an educated reader.

2.1. Interpretational geometry and anomalous virtual realities. Interpretational geometry is a certain geometry related to interactive computer-graphic psychoinformation systems. Mathematical data in such systems exist in the form of an interrelation of an interior geometric image (figure) in the subjective space of observer and an exterior computer-graphic representation; the least includes the visible elements (draws of figure) as well as of the invisible ones (f.e. analytic expressions and algorythms of the constructing of such draws). Process of the corresponding of a geometrical image (figure) in the interior space of observer to a computer-graphic representation (visible and invisible elements) is called *translation*; the visible object maybe nonidentical to the figure, in this case partial visible elemnts may be regarded as modules, which translation is realized separately; the translation is called by *interpretation* if the translation of partial modules is realized depending on the result of the translation of preceeding ones.

Definition 1. A figure, which is obtained as a result of the interpretation, is called *interpretational figure*; the draw of an interpretational figure is called *symbolic*.

Note that the simbolic draws may be characterized only as "visual perception technology" of figure but not as its "image".

The computer–geometric description of mathematical data in interactive information systems is deeply related to the concept of anomalous virtual reality. It should be mentioned that there exist several approaches to foundations of geometry: in one of them the basic geometric concept is a space (a medium, a field), geometry describes various properties of a space and its states, which are called the draws of figures; it is convenient to follow this approach for the purposes of the describing of geometry of interactive information systems; the role of the medium is played by an anomalous virtual reality, the draws of figures are its certain states.

Definition 2.

A. *Anomalous virtual reality* (*AVR*) *in a narrow sense* is a certain system of rules of non–standard descriptive geometry adopted to a realization on videocomputer (or multisensor system of "virtual reality" [BC, Rh, VR1, VR2, VR3]); *anomalous virtual reality in a wide sense* contains also an image in the cyberspace made accordingly to such system of rules; we shall use this term in a narrow sense below.

B. *Naturalization* is the corresponding of an AVR to an abstract geometry or a physical model; we shall say that the AVR *naturalizes* the model and such model *transcendizes* the naturalizing AVR. *Visualization in a narrow sense* is the corresponding of certain images or visual dynamics in the AVR to objects of the abstract geometry or processes in the physical model; *visualization in a wide sense* also includes the preceeding naturalization.

C. An anomalous virtual reality, whose images depends on an observer, is called *intentional anomalous virtual reality* (*IAVR*); generalized perspective laws in IAVR contain the equations of dynamics of observed images besides standard (geometric) perspective laws; a process of observation in IAVR contains a physical process of observation and a virtual process of intention, which directs an evolution of images accordingly to dynamical laws of perspective.

In the intentional anomalous virtual reality objects of observation present themselves being connected with observer, who acting on them in some way, determines, fixes their observed states, so an object is thought as a potentiality of a state from the defined spectrum, but its realization depends also on observer; the symbolic draws of interpretational figures are presented by states of a certain IAVR.

Note that a difference of descriptive geometry of computer-graphic information systems from the classical one is the presense of colors as important bearers of visual information; a reduction to shape graphics, which is adopted in standard descriptive geometry, is very inconvenient, since the use of colors is very familiar in the scientific visualization [SV1, SV2, Vi1, Vi2]. The approach to the computer-graphic interactive information systems based on the concept of anomalous virtual reality allows to consider an investigation of structure of a color space as a rather pithy problem of descriptive geometry, because such space maybe much larger than the usual one and its structure may be rather complicated.

Definition 2D. A set of continuously distributed visual characteristics of image in an anomalous virtual reality is called *anomalous color space*; elements of an anomalous color space, which have non–color nature, are called *overcolors*, and quantities, which transcendize them in the abstract model, are called *"latent lights"*. *Color–perspective system* is a fixed set of generalized perspective laws in fixed anomalous color space.

2.2. Quantum projective field theory and mobilevision. It seems to be a significant fact that 2D quantum field theory maybe expressed in terms of interpretational geometry, so that various objects of this theory are represented by interpretational figures. The keypoint is *mobilevision* (*MV*), which is an IAVR naturalizing *the quantum projective field theory* (*QPFT*; [BR, Ju3] and refs wherein); the process of naturalization is described in [Ju2, Ju3, Ju4, Ju5].

Let's concentrate our attention on the basic concepts of the QPFT, which naturalization mobilevision is.

Definition 3A. *QFT–operator algebra* (*operator algebra of the quantum field theory, vertex operator algebra, vertex algebra*) is the pair $(H, t^k_{ij}(\vec{x}))$: H is a linear space, $t^k_{ij}(\vec{x})$ is H–valued tensor field such that $t^l_{im}(\vec{x})t^m_{jk}(\vec{y}) = t^m_{ij}(\vec{x}-\vec{y})t^l_{mk}(\vec{y})$.

Let us intruduce the operators $l_{\vec{x}}(e_i)e_j = t_{ij}^k(\vec{x})e_k$, then the following relations will hold: $l_{\vec{x}}(e_i)l_{\vec{y}}(e_j) = t_{ij}^k(\vec{x}-\vec{y})l_{\vec{y}}(e_k)$ (*operator product expansion*) and $l_{\vec{x}}(e_i)l_{\vec{y}}(e_j) = l_{\vec{y}}(l_{\vec{x}-\vec{y}}(e_i)e_j)$ (*duality*). Also an arbitrary QFT–operator algebra one can define an operation depending on the parameter: $m_{\vec{x}}(e_i,e_j) = t_{ij}^k(\vec{x})e_k$; for this operation the following identity holds: $m_{\vec{x}}(\cdot, m_{\vec{y}}(\cdot,\cdot)) = m_{\vec{y}}(m_{\vec{x}-\vec{y}}(\cdot,\cdot),\cdot)$; the operators $l_{\vec{x}}(f)$ are the operators of the left multiplication in the obtained algebra.

Definition 3B. QFT–operator algebra $(H, t_{ij}^k(u); u \in \mathbb{C})$ is called *(derived) QPFT–operator algebra* iff (1) H is the sum of Verma modules V_α over $\mathrm{sl}(2,\mathbb{C})$ with the highest vectors v_α and the highest weights h_α, (2) $l_u(v_\alpha)$ is a primary field of spin h_α, i.e. $[L_k, l_u(v_\alpha)] = (-u)^k(u\partial_u + (k+1)h_\alpha)l_u(v_\alpha)$, where L_k are the $\mathrm{sl}(2,\mathbb{C})$ generators ($[L_i,L_j] = (i-j)L_{i+j}$, $i,j=-1,0,1$), (3) the (derived) rule of descendants generation holds ($[L_{-1}l_u(f)] = l_u(L_{-1}f)$). (Derived) QPFT–operator algebra $(H, t_{ij}^k(u))$ is called *projective G–hypermultiplet*, iff the group G acts in it by automorphisms, otherwords, the space H possesses a structure of the representation of the group G, the representation operators commute with the action of $\mathrm{sl}(2,\mathbb{C})$ and $l_u(T(g)f) = T(g)l_u(f)T(g^{-1})$.

The linear spaces of the highest vectors of the fixed weight form subrepresentations of G, which are called *multiplets* of projective G-hypermultiplet.

Now let's describe the key moments of the process of naturalization of the QPFT which is resulted in MV. Unless the abstract model (QPFT) has a quantum character the images in its naturalization (MV) are classical; the transition from the quantum field model to classical one is done by standard rules, namely, the classical field with Taylor coefficients $|a_k|^2$ is corresponded to the element $\sum a_k L_{-1}^k v_\alpha$ of the QPFT–operator algebra. Under the naturalization three classical fields are identified with fields of three basic colors (red, green and blue), other fields with fields of overcolors; there are pictured only the color characteristics for the fixed moment of time on the screen of the videocomputer as well as the perception of the overcolors by an observer is determined by the intentional character of the AVR of mobilevision. Namely, during the process of the evolution of the image, produced by the observation, the vacillations of the color fields take place in accordance to the dynamical perspective laws of MV (Euler formulas or Euler–Arnold equations). These vacillations depend on the character of an observation (f.e. an eye movement or another dynamical parameters); the vacillating image depends on the distribution of the overcolors, that allows to interpret the overcolors as certain *interactive vacillations* of the ordinary colors. So the overcolors of MV are vacillations of the fixed type and structure of ordinary colors with the defined dependence on the parameters of the observation process; the transcending "latent lights" are the quantized fields of the basic model of the QPFT.

The presence of the SU(3)–symmetry of classical color space allows to suppose that the QPFT–operator algebra of the initial model is the projective SU(3)–hypermultiplet.

2.3. Quantum conformal and q_R–conformal field theories; quantum–field analogs of Euler–Arnold tops.

Definition 4A. The highest vector T of the weight 2 in the QPFT–opeartor algebra will be called *the conformal stress–energy tensor* if $T(u) := l_u(T) = \sum L_k(-u)^{k-2}$, where the operators L_k form the Virasoro algebra: $[L_i,L_j] = (i-j)L_{i+j} + \frac{i^3-i}{12}c\cdot I$. The set of the highest vectors J^α of the weight 1 in the QPFT–operator algebra will be called the set of *the affine currents* if $J^\alpha(u) := l_u(J^\alpha) = \sum J_k^\alpha(-u)^{k-1}$, where the operators J_k^α form *the affine Lie algebra*: $[J_i^\alpha, J_j^\beta] = c_\gamma^{\alpha\beta}J_{i+j}^\gamma + k^{\alpha\beta}\cdot i\delta(i+j)\cdot I$.

If there is defined a set of the affine currents in the QPFT–operator algebra then one can construct the conformal stress–energy tensor by use of Sugawara construction. Below we shall be interested in the special deformations of the quantum conformal field theories in class of the quantum projective ones, which will be called *quantum q_R–conformal field theories*; the crucial role is played by so–called *Lobachevskii algebra* in their constructions. In the Poincare realization of the Lobachevskii plane (the realization in the unit disk) the Lobachevskii metric maybe written as $w = q_R^{-1}\,dzd\bar{z}/(1-|z|^2)^2$; one can construct the C^*–algebra (Lobachevskii algebra), which maybe considered as a quantization of such metric, namely, let us consider two variables t and t^*, which obey the following commutation relations: $[tt^*, t^*t] = 0$, $[t,t^*] = q_R(1-tt^*)(1-t^*t)$ (or in an equivalent form $[ss^*, s^*s] = 0$, $[s,s^*] = (1-q_Rss^*)(1-q_Rs^*s)$, where $s = (q_R)^{-1/2}t$); one may realize such variables by bounded operators in the Verma module over $\mathrm{sl}(2,\mathbb{C})$ of the

weight $h = \frac{q_R^{-1}+1}{2}$ (this relation between h and q_R will be presupposed below); if such Verma module is realized in polynomials of one complex variable z and the action of $\mathrm{sl}(2,\mathbb{C})$ has the form $L_{-1}=z$, $L_0 = z\partial_z + h$, $L_1 = z(\partial_z)^2 + 2h\partial_z$, then the variables t and t^* are represented by tensor operators $D = \partial_z$ and $F = z/(z\partial_z + 2h)$. These operators are bounded if $q_R > 0$ and therefore one can construct a Banach algebra generated by them and obeying the prescribed commutation relations; the structure of C^*–algebra is rather obvious: an involution $*$ is defined on generators in a natural way, because the corresponding tensor operators are conjugate to each other.

Definition 4B. The highest vector T of the weight 2 in the QPFT–operator algebra will be called *the q_R–conformal stress–energy tensor* if $T(u) := l_u(T) = \sum L_k(-u)^{k-2}$, where the operators L_k form the q_R–Virasoro algebra: $[L_i, L_j] = (i-j)L_{i+j}$ $(i,j \geq -1;\ i,j \leq 1)$, $[L_2, L_{-2}] = H(L_0+1) - H(L_0-1)$, $H(t) = t(t+1)(t+3h-1)^2/((t+2h)(t+2h-1))$ (cf.[Ro]). The set of the highest vectors J^α of the weight 1 in the QPFT–operator algebra will be called the set of *the q_R–affine currents* if $J^\alpha(u) := l_u(J^\alpha) = \sum J^\alpha_k(-u)^{k-1}$, where the operators J^α_k form *the q_R–affine Lie algebra*: $J^\alpha_k = J^\alpha T^{-k} f_k(t)$, $[J^\alpha, J^\beta] = c^{\alpha\beta}_\gamma J^\gamma$, $Tf(t) = f(t+1)T$, $[T, J^\alpha] = [f(t), J^\alpha] = 0$, $f_k(t) = t\ldots(t-k)$, if $k \geq 0$, and $((t+2h)\ldots(t+2h-k+1))^{-1}$, if $k \leq 0$.

It should be mentioned that q_R–affine currents and q_R–conformal stress–energy tensor are just the $\mathrm{sl}(2,\mathbb{C})$–primary fields in the Verma module V_h over $\mathrm{sl}(2,\mathbb{C})$ of spin 1 and 2, respectively; if such module is realized as before then $J_k = \partial_z^k$, $J_{-k} = z^k/(\xi+2h)\ldots(\xi+2h+k-1)$; $L_2 = (\xi+3h)\partial_z^2$, $L_1 = (\xi+2h)\partial_z$, $L_0 = \xi + h$, $L_{-1} = z$, $L_{-2} = z^2\frac{\xi+3h}{(\xi+2h)(\xi+2h+1)}$, $\xi = z\partial_z$. So the generators J^α_k of q_R–affine algebra maybe represented via generators of Lobachevskii C^*–algebra: $J^\alpha_k = J^\alpha t^k$, if $k \geq 0$, and $J^\alpha(t^*)^{-k}$, if $k \leq 0$, $([J^\alpha, J^\beta] = c^{\alpha,\beta}_\gamma J^\gamma)$. That means that q_R–affine algebra admits a homomorphism in a tensor product of the universal envelopping algebra $\mathcal{U}(\mathfrak{g})$ of the Lie algebra $\mathfrak{g}$, generated by J^α, and Lobachevskii algebra. The (derived) QPFT–operator algebras generated by q_R–affine currents are called canonical projective G–hypermultiplets. The primary fields $V_k(u) = \exp(k(Q + R(\int V_1(u)\,du)))$ $(R(u^n) = -\operatorname{sgn}(n)u^n$, i.e. R is the Hilbert transform $f(\exp(it)) \mapsto -\frac{i}{2\pi}\int f(\exp(i(t-s)))\cot(s/2)\,ds$; a charge Q is defined as $Q(z^n) = \sum_{j=0}^{n-1}(j+2h)^{-1}z^n$; [BJ1,By]) of non–negative integer spins k in the Verma module V_h, which form a closed QPFT–operator algebra (a subalgebra of $\mathrm{Vert}(\mathrm{sl}(2,\mathbb{C}))$ [BJ2], generated by vertex operator fields $B_k(u;\nabla_h)$ [Ju1]), are not mutually local. It is interesting to calculate T–exponent and monodromy of q_R–affine current; it maybe easily performed by a perturbation of simple formulas for such objects for a singular part of a current, as it was stated in [Ju2] such perturbation by a regular part does not change the resulting monodromy.

Let H be an arbitrary direct sum of Verma modules over $\mathrm{sl}(2,\mathbb{C})$ and P be a trivial fiber bundle over $\mathbb{C}$ with fibers isomorphic to H; it should be mentioned that P is naturally trivialized and possesses a structure of $\mathrm{sl}(2,\mathbb{C})$–homogeneous bundle. A $\mathrm{sl}(2,\mathbb{C})$–invariant Finsler connection $A(u,\dot u)$ in P is called an angular field; angular field $A(u,\dot u)$ may be expanded by $(\dot u)^k$, the coefficients of such expansion are just $\mathrm{sl}(2,\mathbb{C})$–primary fields; the equation $\dot\Phi_t = A(u,\dot u)\Phi_t$, where Φ_t belongs to H and $u = u(t)$ is the function of scanning, is a quantum–field analog of the Euler formulas; such analog describes an evolution of MV image under the observation (more rigorously, such evolution is defined in the dual space H^* by formulas $\dot\Phi_t = A^{\mathrm{T}}(u,\dot u)\Phi_t$). Regarding canonical projective G–hypermultiplet we may construct a quantum field analog of the Euler–Arnold equation $\dot A = \{\mathcal{H}, A_t\}$, where an angular field $A(u,\dot u)$ is considered as an element of the canonical projective G–hypermultiplet being expanded by $\mathrm{sl}(2,\mathbb{C})$–primary fields of this hypermultiplet, $\mathcal{H}$ is the quadratic $\mathrm{SU}(3)$–invariant element of $S(\mathfrak{g})$, $\{\cdot,\cdot\}$ are canonical Poisson brackets in $S(\mathfrak{g})$. It is possible to combine Euler–Arnold equations with Euler formulas to receive the complete dynamical perspective laws of the MV. The main feature of these laws is their *projective invariance*, so they define a natural generalization of ordinary descriptive geometry for an interpretational case. The projective invariance fixes Euler formulas uniquely, whereas it allows, of course, to change Euler–Arnold equations to other ones (hamiltonian or ever nonhamiltonian). However, such equations should provide $\mathrm{SU}(3)$–invariance of the dynamical perspective laws.

2.4. Organizing MV cyberspace. MV cyberspace consists of a space of images V_I with the fundamental length (a step of the lattice) Δ_I and a space of observation V_O with the fundamental length Δ_O; the space of images V_I is one where pictures are formed, whereas the space of observation V_O is used for a detection of eye motions; it is natural to claim that $\Delta_O \ll A_{\mathrm{tr}}$, where A_{tr} is an amplitude of the eye tremor, as well as $\Delta_I \gg \Delta_O$. The Euler formulas maybe written as $\dot\Phi_t = A(u,\dot u)\Phi_t$, $A(u,\dot u)$ maybe considered approximately in the form

$M_1(t)\dot{u}V_1(u) + M_2(t)\dot{u}^2V_2(u) + M_3(t)\dot{u}^3V_3(u)$, where $\Phi_t \in H$ (or in the form $\dot{\Phi}_t = A^{\mathrm{T}}(u,\dot{u})\Phi_t$, $\Phi_t \in H^*$); here $M_i(t)$ are data of Euler–Arnold top, $V_i(u)$ are $\mathrm{sl}(2,\mathbb{C})$–primary fields in the Verma module V_h, which maybe written as $V_i(u) = (-u)^{-i}(W_i(u) + W_i^*(u) - D_0^i) = \sum_{j\in\mathbb{Z}}(-u)^{-i-j}D_j^i$, $W_i(u) = \sum_{j\geq 0}(-u)^{-j}D_j^i$, $W_i^*(u) = \sum_{j\geq 0}(-u)^{j}D_{-j}^i$, $D_{-j}^i = (D_j^i)^*$, where $*$ is the conjugation in the unitarizable Verma module. The tensor operators D_k^i ($k \geq 0$, $i = 1,2,3$) have the form $D_k^i = P_{i,k}(z\partial_z)\partial_z^k$, $P_{1,k}(t) = 1$, $P_{2,k}(t) = t + (k+1)h$, $P_{3,k}(t) = t^2 + ((k+2)h + k/2)t + h(2h+1)(k+1)(k+2)/6$. It should be mentioned that the fields $W_i^{\mathrm{T}}(u)$ in local $\mathrm{sl}(2,\mathbb{C})$–modules V_h^* are defined by rather simple expressions:

$$W_1^{\mathrm{T}}(u) = \frac{1}{1-u^{-1}x}, \qquad W_2^{\mathrm{T}}(u) = -\frac{x}{1-u^{-1}x}\partial_x + \frac{h}{(1-u^{-1}x)^2},$$
$$W_3^{\mathrm{T}}(u) = \frac{x^2}{1-u^{-1}x}\partial_x^2 - (2h+1)\frac{x}{(1-u^{-1}x)^2}\partial_x + \frac{h(2h+1)}{3}\frac{1}{(1-u^{-1}x)^3}.$$

The matrix $A^{\mathrm{T}}(u,\dot{u})$ of size (N,N) (N is a number of points of V_I) should be expanded in a sum of three terms $M_i(t)\dot{u}^iV_i(u)$ ($i = 1,2,3$), where $V_i^{\mathrm{T}}(u)$ are matrices of size (N,N), depending on parameter u; this parameter may have M different values (M is number of points of V_O). Matrices $W_i^{\mathrm{T}}(u)$ are easily calculated, one should obtain the complete matrices $V_i^{\mathrm{T}}(u)$ making a conjugation in the unitary local $\mathrm{sl}(2,\mathbb{C})$–module V_h^*. Derivatives should be replaced by differences everywhere in a standard way. Formulas for $M_i(t)$ maybe received from [MF].

2.5. Non–Alexandrian geometry of mobilevision. It should be marked that almost all classical geometries use a certain postulate, which we shall call Alexandrian, but do not include it in their axiomatics explicitely. A precise formulation of this postulate is given below.

Alexandrian postulate. *Any statement holding for a certain geometric configuration remains true if this configuration is considered as a subconfiguration of any its extension.*

Alexandrian postulate means that an addition of any subsidiary objects to a given geometric configuration does not influence on this configuration. It is convenient to describe a well–known example of non–Alexandrian geometry (which maybe called Einstein geometry).

Example of non-Alexandrian geometry. Objects of geometry are weighted points and lines. Weighted points are pairs (a standard point on a plane, a real number). They define a (singular) metric on a plane via Einstein–type equations $R(x) = \sum m_\alpha\delta(x - x_\alpha)$, where (x_α, m_α) are weighted points and $R(x)$ is a scalar curvature. Lines are geodesics for this metric. The basic relation is a relation of an incidence.

It can be easily shown that Alexandrian postulate doesn't hold for such geometry, which contains a standard Euclidean one (extracted by the condition that all "masses" m_α are equal to 0).

Kinematics and process of scattering of figures maybe illustrated by another important example of non–Alexandrian geometry — geometry of solitons [ZMNP]. The basic objects of KdV–soliton geometry are moving points on a line; a configuration of such points defines a n–soliton solution of KdV–equation $u_t = 6uu_{xx} - u_{xxx}$ by the formulas $u(x,t) = -2(\log(\det(E+C)))_{xx}$, where $C_{nm} = c_n(t)c_m(t)\exp(-(\varkappa_n+\varkappa_m)t)/(\varkappa_n+\varkappa_m)$, $c_n(t) = c_n(0)\exp(4\varkappa_n^3t)$; such solution is asymptotically free, i.e. maybe represented as a sum of 1–soliton solutions (solitons) whereas $t \to \pm\infty$. Soliton has the form $u(x,t) = -2\varkappa^2\cosh^{-2}(\varkappa(x - 4\varkappa^2t - \varphi))$, where phase φ is an initial position of soliton and $v = 4\varkappa^2$ is its velocity; scattering of solitons is two–particle, the shift of phases is equal to $\varkappa_1^{-1}\log|(\varkappa_1+\varkappa_2)|/|(\varkappa_1-\varkappa_2)|$ for the first (quick) soliton and $-\varkappa_2^{-1}\log|(\varkappa_1+\varkappa_2)|/|(\varkappa_1-\varkappa_2)|$ for the second (slow) one. All examples of soliton geometries confirm the opinion that a breaking of the Alexandrian postulate is generated by an interaction of geometrical objects, in particular, such interaction maybe defined by a nonlinear character of their evolution.

Let's consider now an interpretational scattering. As it was stated below a figure in interpretational geometry is described by a pair $(\Phi^{\mathrm{int}}, \Phi^{\mathrm{ext}})$, where Φ^{int} is an interior image in the subjective space of observer and Φ^{ext} is its exterior computer-graphic draw; Φ^{int} is a result of interpretation of Φ^{ext}. It is natural to suppose that Φ^{int} depends on Φ^{ext} functionally $\Phi_t^{\mathrm{int}} = \Phi^{\mathrm{int}}\left[\Phi_{\tau\leq t}^{\mathrm{ext}}\right]$ and as a rule nonlinearly; moreover, if Φ^{ext} is asymptotically free then Φ^{int} is also asymptotically free. Thus, a nontrivial scattering of interacting interpretational figures exists (i.e. although we

do not know an explicit form of dynamical equations for Φ^{int}, their solutions, nevertheless, in view of our assumptions maybe considered as *a priori* soliton–like), so interpretational geometries maybe considered as non–Alexandrian ones; it should be specially marked that the breaking of Alexandrian postulate is realized on the level of figures themselves, but it is not observed on one of their draws.

Informatic aspects of mobilevision are considered in the second part of [Ju5].

III. MATHEMATICAL ASPECTS OF STOCHASTIC MOBILEVISION [Ju8]

3.1. Mathematical set up. First of all, stochastic mobilevison as well as ordinary mobilevision are interactive computer-graphic systems, the evolution of images in which is governed by the eye movements in accordance to the certain *dynamical perspective laws*, i.e. dynamical equations, which govern an evolution of image during observation (see par.II or [Ju4,Ju5]). So their definitions are just the specifications of such laws (it should be specially stressed that we restrict now our interest in interactive computer-graphic systems by an intrinsic *constructive* point of view [KT], considering them *as such* but not as *descriptive* tools of any use for modelling or visualizing of various physical processes (as in [Ju5]), such approach may be rather narrow but effective and it is reasonable to adopt it for the further discussion). The laws for MV were written in par.II (or in [Ju2, Ju3, Ju4, Ju5]). Stochastic mobilevision have the slightly different laws. A difference may be briefly summarized in the following terms: (1) the high–frequency eye tremor is decoupled from the slow eye motions (including saccads), (2) it is stochastized in such a way that it may be considered as *purely internal process* in the system so that (3) its characteristics are not completely determined by the real eye motion and may be reinforced.

This qualitative description of stochastic mobilevision is sufficient for the understanding of results as well as their significance for applications but we need in a more formal definition for their deduction. However, a reader, which is not interested in formal expositions may omit all mathematical constuctions below and restrict himself to some comments.

Note once more that to define stochastic mobilevision means to specify its dynamical perspective laws (dynamical equations, which govern an evolution of image during observation) and we prefer to do it rather formally in purely mathematical terms. Such specification is rather analogous to one for the ordinary MV and is based on concepts of 2D quantum field theory. The interpretations of mathematical results and their significance for applications will be commented in detail throughout the text, in the conclusion and in remarks on applications after it.

Definition 5. Let H be a canonical projective G–hypermultiplet, $A_t(u,\dot u)$ – an angular field (obeying the Euler–Arnold equations $\dot A_t = \{\mathcal{H}, A_t\}$, where the hamiltonian $\mathcal{H} \in S^{\cdot}(\mathfrak{g})$ ($\mathfrak{g}$ is the Lie algebra of a Lie group G) is a solution of the Virasoro master equation) (or its finite–dimensional lattice approximations of par.II or [Ju5]). Let $J(u)$ — an additional q_R–affine current (par.II or [Ju3, Ju4])(or its finite–dimensional lattice approximation from par.II or [Ju5]) commuting with G. A stochastic evolution equation

$$d\Phi(t,[\omega]) = A_t(u,\dot u)\Phi(t,[\omega])\,dt + \lambda J(u)\Phi(t,[\omega])\,d\omega,$$

where $d\omega$ is the stochastic differential of a Brownian motion (i.e. $\frac{d\omega}{dt}$ is a white noise), will be called *the (quantum–field) Euler–Belavkin–Kolokoltsov formulas*, the parameter λ will be called *the accuracy of measurement* (cf. [Be, BK, Ko1]).

Remark 1. These formulas are a reduced version of more general ones

$$d\Phi(t,[\omega])=\{A_t(u,\dot u)+\alpha\lambda^2\ :J^2(u)\!:\}\Phi(t,[\omega])\,dt+\lambda J(u)\Phi(t,[\omega])\,d\omega,\qquad(\alpha>0)$$

which will be also called *the (quantum–field) Euler–Belavkin–Kolokoltsov formulas*; $\lambda^2\ :J^2(u):$ is a Belavkin–type quantum–field counterterm (cf. [Be, BK, Ko1, Ko2]), where $:J^2(u):$ is a spin–2 primary field received from the current $J(u)$ by the truncated Sugawara construction [Ju3].

Here $u=u(t)$ and $\dot u=\dot u(t)$ are the slow variables [Ju2] of observation (sight fixing point and its relative velocity), the tremor is simulated by a stochastic differential $d\omega$, λ is a *free* parameter, $\Phi=\Phi(t,[\omega])\in H$ is a collective notation for a set of all continuously distributed characteristics of image [Ju2, Ju4, Ju5], q_R is a free internal parameter of a model, which measures the degree of localization of interaction (the local case corresponds to $q_R=0$). The most important case is one of $q_R\ll 1$ and all our results will hold for this region of values of q_R. The stochastic Euler–Belavkin–Kolokoltsov formulas coupled with the deterministic Euler–Arnold equations define a dynamics, which may be considered as *a candidate* for one of *a continuously observed (and interactively controlled) quantum–field top* [Ju3].

Remark 2. It should be specially emphasized that in stochastic mobilevision λ is a *free* parameter, which may be chosen arbitrary by hands (f.e. as great as it is necessary). It means that slow movements (including saccads) and tremor are decoupled, the firsts are considered such as in an ordinary MV, whereas the least is stochastized in a way that *its amplitude may be reinforced*.

Remark 3. As it was mentioned above the internal parameter q_R measures a degree of localization of a man–machine interaction in MV and SMV. It is natural to suppose that the Belavkin–Kolokoltsov watch–dog effects will appear for sufficiently small values of q_R and the condition $q_R\to 0$ will produce the diminishing of stochastic cores of image. Indeed, we shall see that sizes of stochastic cores diminish if q_R tends to 0 and λ increases.

Below we shall work presumably with finite–dimensional lattice approximations (cf. [Ko2]) and the associate evolution equation in H^* (see par.II or [Ju5]), keeping all notations. Also Φ will be considered as defined on a compact (the screen of a display or a cluster). It should be marked that in this case the Euler–Belavkin–Kolokoltsov formulas are transformed into the ordinary (matrix) stochastic differential equations of diffusion type [GS, Sk], and hence, $\Phi=\Phi_t=\Phi(t,[\omega])$ is a diffusion Markov process [Dy].

Remark 4. Lattice approximations of the ordinary (unobserved and non–controlled) quantum–field top (in this case angular fields are reduced to single currents) were actively investigated by St.Petersburg Group directed by Acad.L.D.Faddeev [AFS]. The main difficulties (technical as well as principal) in their treatments were caused by a locality of ordinary ($q_R=0$) affine currents. However, q_R–affine currents are not local so their discretizing is easily performed (see par.II or [Ju5]). It is very interesting to receive lattice current algebras of [AFS] from naturally discretized q_R–affine currents by a limit transition $q_R\to 0$, but this problem is a bit out of the line here.

The fact that the ordinary quantum–filed top may be received as a particular case of our construction ($\lambda = 0$, $q_R = 0$, $A(u, \dot{u}) = J(u)\dot{u}$, where $J(u)$ is a current) motivates to consider our object as a continuously observed (and interactively controlled) quantum–field top. Continuous observation means the inclusion of a stochastic term ($\lambda \neq 0$), whereas the interactive controlling means the presence of complete algular fields $A(u, \dot{u}) = \sum_k B_k(u)\dot{u}^k$, where $B_k(u)$ are primary fields of spin k, instead of single currents. It seems that these arguments are sufficient for our terminological innovation.

Remark 5. The Euler–Belavkin–Kolokoltsov formulas are *postulated* to be the dynamical perspective laws for stochastic mobilevision. So they are regarded as *a mathematical definition of SMV*. From such point of view a transition from MV to SMV consists in:

1) the decoupling of slow movements (including saccads) and tremor;
2) a stochastization of tremor;
3) the setting the controlling parameter λ free, so that its value may be chosen by hands and it is not completely determined by real parameters of the eye motions.

Thus, the main difference between MV and SMV is that tremor in MV is *an external process* governing an evolution of a computer graphic picture, whereas its stochastization is *an internal process* (in spirit of *endophysics* of Prof. O.E. Rössler [Rö, En]) and its characteristics may be specified by hands.

Let's summarize the material of par.3.1. Note once more that the ordinary mobilevision is an interactive computer-graphic system, the evolution of images in which is governed by the eye movements in accordance to the certain dynamical perspective laws, which were written in par.II or [Ju2, Ju3, Ju4, Ju5]. Stochastic mobilevision is an analogous interactive computer-graphic system, but with slightly different dynamical perspective laws. Namely, in the dynamical perspective laws of MV the high–frequency eye tremor is decoupled from the slow eye motions (including saccads), is stochastized in such a way that it may be considered as *purely internal process* in the system so that its characteristics are not completely determined by eye motions and may be reinforced. So the parameters of an external real process (eye tremor) may be *transformed and scaled up* to receive ones of an internal virtual process (stochastization of tremor). For the understanding of results the explicit form of dynamical perspective laws is not necessary though it is, of course, unavoidable for their deduction, which is presented in par.3.2., which may be omitted by a reader interested only in applications, who may restrict himself by the comment and remark at its end.

3.2. Mathematical analysis. Let $D_A(\Phi) = \left\langle A^2 - \langle A\rangle_\Phi^2 \right\rangle_\Phi$, $\langle A\rangle_\Phi = \frac{(A\Phi,\Phi)}{(\Phi,\Phi)}$ (Kolokoltsov 1993). It should be mentioned that one may consider the Euler–Belavkin–Kolokoltsov formulas with a redefined quantum field $\tilde{J}(u) = J(u) - \langle J(u)\rangle$ instead of the q_R–affine current $J(u)$ to receive a full likeness to the original Belavkin quantum filtering equation [Be, BK, Ko1, Ko2] if the inner (scalar) product $(\cdot,\cdot)$ is claimed to be translation invariant and scaling homogeneous. E_Φ is the mathematical mean with respect to the standard Wiener measure for observation process with initial point Φ [Ko2].

Lemma 1.

$$(\forall\Phi_0)\quad \limsup_{t\to\infty} E_{\Phi_0} D_J(\Phi(t,[\omega])) = K\lambda^{-2} \longrightarrow_{\lambda\to\infty} 0.$$

The l.h.s. expression (multipled by λ^2, i.e. just the constant K) is called *the Kolokoltsov coefficient* of quality of measurement [Ko2].

Sketch of the proof. Indeed

$$\lambda^2 \limsup_{t\to\infty} E_{\phi_0} D_J(\Phi(t,[\omega])) = \limsup_{t\to\infty} E_{\Phi_0} D_{\lambda J}(\Phi(t,[\omega])) = \limsup_{t\to\infty} E_{\widetilde{\Phi}_0} D_J(\widetilde{\Phi}(t,[\omega])),$$

where $\widetilde{\Phi}$ is a solution of the Euler–Belavkin–Kolokoltsov formulas with $\lambda = 1$ and with the initial data $\widetilde{\Phi}_0$ being equal to Φ_0 scaled in λ times (the least equality follows from the scaling homogenity of the Euler–Belavkin–Kolkol'tsov formulas). As a sequence of results of [Ko2](the dependence of the q_R–affine current J on u is not essential in view of the translation invariance) the expression $\limsup_{t\to\infty} E_{\widetilde{\Phi}_0} D_J(\widetilde{\Phi}(t,[\omega]))$, being the Kolokoltsov coefficient $\kappa(A_t, J)$ for the pair (A_t, J), does not depend on $\widetilde{\Phi}_0$, and hence, it is certainly independent on λ.

Remark 6. The sketch of the proof is rather instructive itself. Instead of difficult calculations of the stationary probability measure (cf. [Ko2]) and a complicated estimation of its λ–behaviour (that is non–trivial to perform rather in the simplest 2–dimensional case considered in [Ko2]) we use general group–theoretical properties (the translation invariance and the scaling homogenity) of the Euler–Belavkin–Kolokoltsov formulas, combining them with the strong results of [Ko2] on an existence of the Kolokoltsov coefficient $K = \kappa(A_t, J)$ and its independence on the initial data.

Comments on the proof. Concerning the sketch of the proof two remarks on some details should be made. First, in view of the dependence of the angular field $A_t(u,\dot{u})$ on the controlling parameters the unique stationary probability measure does not exist; however, we consider all controlling parameters as slow ones so one may *assume* that there exists the slowly evoluting stationary probability measure, which form depends only on the current values of controlling parameters (of course, it is clear that such assumption is natural from mathematical physics point of view, however, it means a certain *"gap"* in the rigorous proof from pure mathematics one; but here any *"purification"* will be out of place). Such parameters varies through a compact set (in the continuous version, or may have only finite number of values in the lattice version), so one can define the Kolokoltsov coefficient as the supremum of such coefficients calculated for the measures from the compact (or finite) set (just this circumstance causes the appearing of "$\limsup$" in Lemma 1). However, second, now one may use the scaling rigorously only for infinite regions, whereas we have to deal with finite ones (the screen of a display or clusters); however, the transition to the compact regions may only cause that the Kolokoltsov coefficient K being a function of λ decreases if λ tends to infinity.

Let's Q be the coordinate operator $Qf(x) = xf(x)$; J° be a singular part of the current J (par.II or [Ju2, Ju5]), i.e. $J^\circ(u) = (Q-u)^{-1}$.

Lemma 2.

$$E_{\Phi_0}\left(D_J(\Phi(t,[\omega])) - D_{J^\circ}(\Phi(t,[\omega]))\right) \rightrightarrows 0 \quad \textit{if} \quad q_R \to 0.$$

It should be marked that the statement of the lemma naïvely holds only in the continuous version; after a finite–dimensional approximation the expression "$\rightrightarrows 0$" should be understand as the l.h.s. becomes uniformely less than a sufficiently small constant ϵ (which depends on the chosen approximation), when q_R tends to zero.

Hint to the proof. The lemma follows from the explicit computations of eigenfunctions of a q_R–conformal current $J(u)$.

Main Theorem.

$$(\forall \Phi_0) \qquad \lim_{\lambda\to\infty, q_R\to 0} \limsup_{t\to\infty} E_{\Phi_0} D_Q(\Phi(t,[\omega])) = 0.$$

The statement of the theorem is a natural sequence of two lemmas above; it remains true in the multi–user mode [Ju6] also. Certainly, the statement of the theorem naïvely holds only in the continuous version (cf. Lemma 2); after a finite–dimensional approximation the equality of the limit to 0 should mean that this limit is less than a sufficiently small constant ϵ, which depends on the chosen approximation.

Comment. Thus, we received that the Belavkin–Kolokoltsov watch–dog effects in stochastic mobilevision appear for all values of the accuracy of measurement λ for sufficiently small values of parameter q_R. Moreover, if λ increases and q_R tends to 0 the stochastic cores may be diminished to several pixels.

Remark 7. Note that the Belavkin–Kolokoltsov watch–dog effects appear only in the models of SMV with sufficiently small values of the internal parameter q_R, which measures the localization of interaction ($q_R = 0$ mens the local case). However, q_R, being an internal parameter, may be chosen in arbitrary way, so the condition $q_R \ll 1$ may be always provided.

IV. CONCLUSION

4.1. Summary of results. Thus, the results may be briefly summarized.

First, let's emphasize once more that the main difference of SMV from the ordinary MV is that the stochastization of eye tremor in the first is considered as an internal process, so its amplitude characteristics may be *reinforced*. Second, for *all values* of λ (a free parameter of such stochastization, which measures the reinforcing of the amplitude of tremor — the so–called accuracy of measurement) the Belavkin–Kolkoltsov watch–dog effects for stochastic dynamics of image in SMV are observed (it means that stochastic cores of image have finite sizes for all times) for sufficiently small values of an additional internal parameter q_R; it confirms the presence of watch–dog effects also in the models of ordinary MV with the same q_R. Moreover, third, if the value of λ is great enough, whereas $q_R \ll 1$ than the stochastic scores of SMV image may be diminish to several pixels. Such effect, which is produced by the reinforcing of λ, may be effectively used in practical computer-graphics for various purposes as it was marked in the introduction. Some further discussions of significance of the obtained results for other applications may be found in par.4.2.

4.2. Remarks on applications. Besides theoretical importance for the interactive visualization of 2D quantum field theory the results of the paper seems to be useful for applications to (**1**) the elaboration of computer-graphic interactive systems for psychophysiological self–regulation and cognitive stimulation [Ju4, Ju5], (**2**) the interactive computer-graphic modelling of a "quantum computer" [Ju5] (see [D, Jo, DJ] for a general discussion on "quantum computers" and their use for rapid computations as well as [Un] on fundamental difficulties to construct the "physical" non-interactive "quantum computer"), which may be used for an actual problem of the accelerated processing of the complex sensorial data in the "virtual reality" (visual–sensorial) networks, (**3**) the creation of computer graphic networks of teleæsthetic communication [Ju5].

Let's discuss a significance of obtained results for these applications.

Comment: Obtained results and applications.

(**2**) is directly related to our results because the maintaining of the coherence is the main problem for "quantum computers". As it was mentioned earlier [Ju5] MV may be regarded as an interactive computer-graphic simulation of a "quantum computer" behavior. The presence of free parameters (such as λ) in SMV allows to maintain the coherence for long times with an arbitrary precision in the interactive mode.

Moreover, such interactive computer-graphic simulations may be more useful than the original "quantum computers" for the "virtual reality" problems in view of the implicit presence of graphical datain the interactive mode. A reorganization of these data by the secondary image synthesis [Ju7] and their representation via MV or SMV may allow an accelerated parallel processing of the complex sensorial data in such systems.

(**1**) and (**3**) are indirectly related to our results because they depend on a solution of the main problem of dynamics in interactive psychoinformation computer-graphic systems (a problem of the nondemolition of images). For (**3**) its solution allows to transmit the graphically organized information without a dissipation and additional errors. For (**1**) its solution allows to consider a long–time self–organizing interactive processes, which play a crucial role in systems for psychophysiological self–regulation and cognitive stimulation.

So it should be stressed that the obtained results are essential for the prescribed applications.

4.3. Remarks on generalizations and perspectives. Now let's discuss the possible generalizations.

Really one consider a random (discrete) simulation of the continuous Brownian motion and stochastic differentials. It may be rather interesting to replace it by any their perturbation (f.e. by some version of the weakly self–avoiding or self–attracting walks, especially by their finite memory approximations).

First, these generalizations are motivated by the fact that Brownian motion may be not the best stochastization of the eye tremor. Really, it may be considered only as a first approximation for tremor, whereas the more complicated models will be preferable. However, it seems that the watch–dog effects are conserved by any form of the weakly self–attracting perturbations, which are the most realistic candidates for tremor.

Second, it seems to be rather interesting to use the decoupling of high–frequency tremor from slow eye movements (including saccads) and an internal character

of its stochastic simulations for the organization of various "intelligent" forms of human–computer interaction (the so–called *"semi–artificial intelligence"*). In such approach the stochastized tremor plays a role of an internal observer (cf. [Rö, En]), which presence is crucial for a self–organization of graphical data in systems of the semi–artificial intelligence [KT]. But this topic (though being related to (**1**) above) seems to be too manysided and too intriguing that this paper is not a suitable place to discuss it further.

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